Metameric Proclivity

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Summary

- CIELAB is flawed
- Does not accurately predict illuminant changes
- A cone-based color system would do better
- A metric to assesses color changes under different illumination.
Which to pick?

All have $L^*a^*b^*$ = (70, -5, -5) under D50/2
Steve Smiley’s recommendation

- Pick smooth spectra
- Less inclination to metamerize
- How to measure smoothness?

- I have a different suggestion
- But first a long digression
Now for the first digression

Tristimulus functions
Versus
Cone functions
Are these the cone functions?
Tristimulus functions

- Cornerstone of color measurement today
- CIELAB is based on these
- Developed and standardized by the CIE in 1931
The actual cone functions
History lesson digression

Ummmm....
Why don’t we use the cone functions?
Approximations were known 100 years ago

Troland, Luther T., *Report of committee on colorimetry for 1920-21*, JOSA Vol VI, Number 6, Aug 1922
Various Color Matching Functions from Judd

"... convenience in discussion of psychophysiological theory may justify the temporary adoption of any one of a wide range of sets of distribution curves."

Hardy agreed

“Any set of primaries is adequate, and the advantage of one set over any other set reduces merely to a matter of convenience.”

Hardy, Arthur C., *Handbook of Colorimetry*, Technology Press, 1936
How did the CIE choose?

- Prevailing notion in 1931
  - *It doesn’t matter which CMF is used*
- The committee developed a set of priorities
- Accurately capturing the cone functions was not one of them.
- Computational steps were a big concern!
2020 hindsight is 20/20

“We have shown that likely none of these formulating principles would be adopted if the system were formulated from a fresh start today.”

How the CIE 1931 color-matching functions were derived from Wright-Guild data
Hugh S. Fairman, Michael H. Brill, Henry Hemmendinger,
Color Research and Application, Dec 6, 1998
Another digression
Adams’ original idea for a color space (1923)

Adam’s depiction of cone functions

Fig. 32.—Fundamental sensations on arbitrary scale, equal-area curves measured with sunlight by König and Dieterici.

L—cone vision blue
M—rod vision
N—cone vision green
O—cone vision red (recalculated)
Adams’ revised model

Comparison of Adams’ formula with CIELAB

\[ L\downarrow A = V\uparrow - 1 \ (Y) \]
\[ a\downarrow A = k\downarrow 1 \ (V\uparrow - 1 \ (X) - V\uparrow - 1 \ (Y)) \]
\[ b\downarrow A = k\downarrow 2 \ (V\uparrow - 1 \ (Z) - V\uparrow - 1 \ (Y)) \]

where

\[ V(\mathcal{R}) = 10 \sqrt{1.4743 \mathcal{R} - 0.004743 \mathcal{R}^{1/2}} \]

\[ L\uparrow* = 116 f(Y/Y\downarrow n ) - 16 \]
\[ a\uparrow* = 500 (f(X/X\downarrow n ) - f(Y/Y\downarrow n )) \]
\[ b\uparrow* = 200 (f(Y/Y\downarrow n ) - f(Z/Z\downarrow n )) \]

where

\[ f(q) = q^{1/3} \]
\[ \text{if } q > \left( \frac{24}{116} \right)^{1/3} \]

\[ f(q) = \left( \frac{841}{108} \right) q + 16/116 \]
\[ \text{if } q \leq \left( \frac{24}{116} \right)^{1/3} \]
Three differences

- Different nonlinear function and scaling
- CIELAB scales X, Y, Z to account for different illuminants
- $b_A$ and $b^*$ are inverted from each other
Why isn’t CIELAB computed from cone functions?

- The prevailing attitude was “it doesn’t matter which CMF to use”
- CIE went with tristimulus functions in 1931
- Adams followed suit in 1942
- CIELAB was based on Adams color space

→ There is a flaw in CIELAB
Yet another digression
Concept of chromatic adaptation

- An apple is always red, independent of the lighting.
- Despite the fact that
  - Light levels change by orders of magnitude
  - Incandescent light has about one-tenth of the intensity at the blue end but D50 is rather flat
- The cones are auto-ranging
- The *cones* are auto-ranging, not the tristimulus functions!
Another look at the cone functions

Trimstimulus Functions

400 450 500 550 600 650 700
The prevailing attitude was “it doesn’t matter which CMF to use’’.
CIE went with tristimulus functions in 1931.
Adams followed suit in 1942.
CIELAB was based on Adams color space.

→ There is a flaw in CIELAB ←
Chromatic adaptation in CIELAB

8.2.1.1 Basic coordinates

Three-dimensional, approximately uniform coordinates, $L^*$, $a^*$, $b^*$, quantities defined

\[
L^* = 116 \left( \frac{Y}{Y_n} \right)^{1/3} - 16
\]
\[
a^* = 500 \left( \frac{X}{X_n} \right) - \left( \frac{Y}{Y_n} \right) - \left( \frac{Z}{Z_n} \right)
\]
\[
b^* = 200 \left( \frac{Y}{Y_n} \right) - \left( \frac{Z}{Z_n} \right)
\]

We normalize by dividing by something that doesn’t exist in the eye/brain.
Chromatic adaptation via Bradford transform in XYZ

- Step 1: convert from XYZ to LMS*
- Step 2: scale LMS for light source
- Step 3: convert back to XYZ

*XYZ to LMS – Multiply by Bradford matrix
Chromatic adaptation via Bradford transform in L*a*b*

- Step 0: convert from L*a*b* to XYZ
- Step 1: convert from XYZ to LMS
- Step 2: scale LMS for light source
- Step 3: convert back to XYZ
- Step 4: convert from XYZ to L*a*b*

Seven out of five color scientists don’t recommend this!
Chromatic adaptation via Bradford transform in color space based on cone functions

- Step 1: convert from $L_c a_c b_c$ to LMS
- Step 2: scale LMS for light sources (Multiply LMS by 1)
- Step 3: convert LMS back to $L_c a_c b_c$

Chromatic adaptation would have been automatic if Adams had stuck to his 1923 concept!!
In retrospect, CIELAB should have been based on LMS rather than XYZ,
since CIELAB will distort color when you convert to a different illuminant.
Color constancy

- Defined
  - *Apples stay red under different illuminants*
- Good for brand owners
- Good for printers…
Limits to color constancy
Limits to color constancy
Metamerism in color matching

- Printer is given L*a*b*
- Uses color mixing software to find a formulation
- Best for printer to go with formulation with best color constancy
- How to do that?
Yet another digression????

- This may sound like a sidetrack
- And it is
- But it’s not
CIELAB predictions of color change

- I started with 81 grid points in CIELAB D50/2
- All with L*=50
- Converted to D65/2
- Used the Bradford transform for chromatic adaptation

- *Five out of four color scientists don’t recommend using the Bradford transform on CIELAB*
- Some people do it anyway
CIELAB predicted color change (D50/2 to D65/2)
Possible explanations

- 1. This could be true color inconstancy.
- 2. This could be an artefact of bad normalization
- 3. Maybe metamerism?
Enter my metameric database.

I have created a database of 38K metameric sets, all real world, from seven different printing modalities.
Metameric septuplets - L*a*b* of (50, 40, 40)
They are metamers under D50/2, not D65/2
To avoid confusion

- Bradford chromatic adaptation
  - estimates the change in color
  - uses $L^*a^*b^*$, not the spectrum
- In the previous slide, I am computing from spectral data
They are metamers under D50/2, not D65/2
They are metamers under D50/2, not D65/2
Change in color of D50 metamers to D65
This is systematic, kinda consistent with Bradford
This is systematic, kinda consistent with Bradford
Possible explanations

- 1. This is true color inconstancy.
- 2. This is a tristimulus-induced artefact.
- 3. Metamerism?

Metamerism doesn’t explain it all.
What if we went back to 1923, with full knowledge of LMS, and full knowledge of the problems that would be created, and created ConeLab as Adams originally conceived?
ConeLab

- Based on LMS cone functions rather than XYZ
- Math is similar
The analog of the tristimulus functions

Bradford transform version of LMS
Comparison of ConeLab with CIELAB

\[ L↓c = 25(0.43g(L/L↓n) + 0.52g(M/M↓n) + 0.52g(S/S↓n)) \]
\[ M↓n = 116 f(Y/Y↓n) - 16 \]
\[ a↑* = 500(f(X/X↓n) - f(Y/Y↓n)) \]
\[ b↑* = 200(f(Y/Y↓n) - f(Z/Z↓n)) \]

where

\[ g(x) = \log_e(20x + 1) \]

\[ f(q) = \frac{q^{1/3}}{24/116} \]

if \( q > \left( \frac{24}{116} \right)^{1/3} \)

\[ f(q) = \left( \frac{841}{108} \right)q + \frac{16}{116} \]

if \( q \leq \left( \frac{24}{116} \right)^{1/3} \)
Comparison of ConeLab with CIELAB

\[ L_{\downarrow c} = 25 \left( 0.43 g(L/L_{\downarrow n}) + 0.52 g(M/M_{\downarrow n}) + 0.05 g(S/S_{\downarrow n}) \right) \]
\[ M_{\downarrow n} + 0.52 g(M/M_{\downarrow n}) \]
\[ a_{\downarrow c} = k_{\downarrow a} (g(L/L_{\downarrow n}) - g(M/M_{\downarrow n})) \]
\[ b_{\downarrow c} = k_{\downarrow b} (g(M/M_{\downarrow n}) - g(S/S_{\downarrow n})) \]

where
\[ g(x) = \log_e (20x+1) \]

\[ f(q) = \begin{cases} q^{1/3} & \text{if } q > \left( \frac{24}{116} \right)^{1/3} \\ \frac{841}{108} q + \frac{16}{116} & \text{if } q \leq \left( \frac{24}{116} \right)^{1/3} \end{cases} \]

For this paper, \( k_{\downarrow a} = 70 \) and \( k_{\downarrow b} = 45 \) were selected to provide a color space of roughly the same size as \( a^* b^* \) at \( L^* = 50 \) in the range of \(-40 \leq a^*, b^* \leq 40\).
Comparison of ConeLab with CIELAB

\[ L_{c} = 25 (0.43 g(L/L_{n}) + 0.52 g(M/M_{n}) + 0.05 g(S/S_{n})) \]
\[ a_{c} = k_{a} (g(L/L_{n}) - g(M/M_{n})) \]
\[ b_{c} = k_{b} (g(M/M_{n}) - g(S/S_{n})) \]

where

\[ g(x) = \log_{\frac{20}{116}} (20x + 1) \]

\[ f(q) = \begin{cases} \frac{q^{1/3}}{24/116} & \text{if } q > \left( \frac{24}{116} \right)^{1/3} \\ \frac{841/108}{q + 16/116} & \text{if } q \leq \left( \frac{24}{116} \right)^{1/3} \end{cases} \]

\[ f(q) = \begin{cases} \frac{q^{1/3}}{24/116} & \text{if } q > \left( \frac{24}{116} \right)^{1/3} \\ \frac{841/108}{q + 16/116} & \text{if } q \leq \left( \frac{24}{116} \right)^{1/3} \end{cases} \]

\( g(x) \) was introduced in Seymour, John, Working Toward a Color Space Built on CIEDE2000, Technical Association of the Graphic Arts, March 2015. This makes DL agree with DE2000.
Color inconstancy (D50/2 to D65/2) in ConeLab

A rotation was added to put red at the wrong angle.
Comparison of CIELAB and ConeLab
Comparison of size of color shifts (38K spectra)
Comparison of size of color shifts

- Median color shift in CIELAB: $3.21 \Delta E_{ab}$
- Median color shift in ConeLab: $1.18 \Delta E_{ab}$

- Roughly two thirds of the color shift when we convert D50 L*a*b* to D65 is caused by an error made in 1931 and propagated in 1942
Very important conclusion

Roughly two thirds of the color shift when we convert D50 L*a*b* to D65 is caused by an flaw in CIELAB made in 1931 and propagated in 1942.
Metameric proclivity

- ConeLab shows considerably more color constancy than CIELAB under a change from D50 to D65.
- How about we use this to measure the degree that a color might be susceptible to metamerism?
Metameric proclivity

- Compute ConeLab values from spectrum under D50, using the Bradford LMS functions.
- Compute ConeLab values from spectrum under D65, using the Bradford LMS functions.
- Determine the color difference.
- $M_p$ is this difference
Which color difference formula?

- I dunno
- ΔE_{00} is preferred for CIELAB
- I have not done any testing of color difference formulas for ConeLab
- L_c already has ΔE_{00} baked in
- M_p is used to compare one metamer to another, and not necessarily to our perception of color
- For the time being use ΔE_{ab}
Summary

- There is a flaw in CIELAB.
- The flaw causes an apparent color inconstancy under change of illuminant.
- Color spaces based on cone functions avoid this flaw.
- This provides a way to test a spectrum for its proclivity to change color under different illuminants.
- I propose $M_p$ as a measure of this.
Thank you
for your attention!

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