

Micro-reflectance Measurements of Multiple Colorants in Halftone Prints

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Abstract

Modeling color reproduction in halftone prints is difficult, mainly because of light scattering in the substrate, causing optical dot gain. Most available models are limited to macroscopic color measurements, averaging the reflectance over an area that is large relative the halftone dot size. The reflectance values for the full tone ink and the unprinted paper are used as input, and these values are assumed to be constant. An experimental imaging system, combining the accuracy of color measurement instruments with a high spatial resolution, allows us to measure the individual halftone dots, as well as the paper between them. Microscopic reflectance measurements reveal that the micro-reflectance of the printed dots and the paper between them is not constant, but varies with the dot area coverage. By incorporating the varying micro-reflectance values of the ink and paper in an expanded Murray-Davies model, we have previously shown that the resulting prediction errors are smaller than for the famous Yule-

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Nielsen model. Moreover, unlike Yule-Nielsen, the expanded Murray-Davies model takes into account the varying micro-reflectance for the printed dots and the paper, thus providing a better physical description of optical dot gain in halftone reproduction.

In this study, we further extend the methodology to handle color prints, predicting tristimulus values for prints with multiple and overlapping colorants. After converting the microscopic images of halftone prints into CIEXYZ color space, 3D histograms are computed. In the 3D histograms, the paper and the inks appear as clusters, with the transitions between the clusters corresponding to the edges of halftone dots. The tristimulus values for the paper and the different combinations of ink are computed as the centers of gravity for the clusters in the 3D histogram. From the microscopic images we can also compute the physical dot area coverage for each of the Neugebauer primaries, which typically differ from the nominal one, due to physical dot gain. The result is an expanded Neugebauer model, employing the varying tristimulus values of the paper and primary inks, as well as for overlapping, secondary colors. Experimental results confirm the accuracy of the proposed methodology, when compared to measurements using a spectrophotometer. Further, the results have shown that the variation of the micro-reflectance of the Neugebauer primaries is large, and depends strongly on the total dot area coverage.

Introduction

To accurately predict the outcome of halftone prints is essential in order to achieve a consistent color reproduction. The task is difficult, partly because of light scattering within the halftone image, causing optical dot gain, also known as the Yule-Nielsen effect. Further more, the optical dot gain often co-exists with physical dot gain, caused by physical dot extension in the printing process. The first model to predict the output reflectance of a monochrome halftone print is the *Murray-Davies* model, published in the 1930s (Murray, 1936). The mean

reflectance R , is simply given by linear interpolation of the reflectance of the bare paper, R_p , and the full tone, R_i , weighted by the dot area fraction, a , as:

$$R(a) = aR_i + (1 - a)R_p \quad (1)$$

The famous *Neugebauer* model (Neugebauer, 1937) is then a relatively straightforward extension of the Murray-Davies formula to handle multiple colorants in color printing. The reflectance is given as the summation of the reflectance for the different colorants (including overlapping colorants), weighted by their fractional area coverage, as:

$$R = \sum_j a_j R_j \quad (2)$$

where a_j is the fractional area for each of the Neugebauer primaries, with reflectance R_j . For a three-color print, the eight Neugebauer primaries correspond to: white (the bare substrate), cyan, magenta and yellow (the primary inks) red, green and blue (the secondary colors) and three color black. Since the reflected light from different areas is added to predict the overall reflectance, these models preserve the linearity of photon additivity. It is, however, well known that the performance of these linear models is very limited. The relationship of R versus a is in fact non-linear, due to light scattering in the paper substrate, causing optical dot gain.

In the 1950s, Yule and Nielsen published their famous work on light penetration and scattering in paper (Yule & Nielsen, 1951). It was then shown that the nonlinear relationship could be approximated by a power function, as:

$$R(a) = \left[aR_i^{1/n} + (1 - a)R_p^{1/n} \right]^n \quad (3)$$

The Yule-Nielsen *n-factor*, accounting for light scattering in the paper, is an empirically derived constant, selected to provide the best fit to experimental data. Unless other factors than optical scattering of light are involved, values of *n* between 1 and 2 are physically meaningful, with *n*=2 corresponding to a highly scattering substrate. The Neugebauer formula has later been combined with the Yule-Nielsen formula, as proposed by Yule and Colt (1951) in terms of print density, and by Viggiano (1985) in its spectral reflectance form. The Yule-Nielsen modified Neugebauer is commonly referred to as the *n-modified Neugebauer* (Emmel, 2003). The Yule-Nielsen and Yule-Nielsen modified Neugebauer models are still commonly used, because they work relatively well. However, they do not physically describe or explain the phenomenon of optical dot gain, and the conservation of energy is lost when the nonlinear transform is applied to the reflectance values. Notice that the fundamental assumption in these models is that the color for the substrate and the ink is both uniform and constant.

In the 1990s, it was shown that the color of the halftone dots and the paper between the dots is not constant, but dependent on the dot area fraction, *a* (Engeldrum, 1994). The reflectance of the printed halftone dots, as well as the paper between them, decreases with increased dot area coverage, due to the light scattering in the substrate. An expanded Murray-Davies model was later proposed, with the constants for paper and ink reflection replaced by the functions $R_i(a)$ and $R_p(a)$ (Arney, et al., 1995a, 1995b):

$$R(a) = aR_i(a) + (1 - a)R_p(a) \quad (4)$$

This model preserves the linear additivity of reflectance while the non-linear relation between R and a caused by optical dot gain, is now accounted for by using the functions $R_i(a)$ and $R_p(a)$. Naturally, the difficulty with this approach is to derive $R_i(a)$ and $R_p(a)$, i.e. the way that the reflectance of the ink and paper shift with varying dot area fraction. Since it is not possible to derive these components using macroscopic measurements, giving only averaged reflectance of the print, there is clearly a need for micro-reflectance measurements of the halftone dots and the paper between them.

In previous work, we have used an experimental imaging system, combining the accuracy of color measurement instruments with a high spatial resolution, to capture microscopic images of halftone prints. From histogram data from such images, $R_i(a)$ and $R_p(a)$, were measured and characterized. The validity of the expanded Murray-Davies model (Eq. 4) was evaluated also for today's high resolution prints, and an extension to tristimulus values were proposed, using 3D histograms in CIEXYZ color space, for single color prints (Nyström, 2008a). The results have shown that the resulting prediction errors are smaller than for the Yule-Nielsen model. Moreover, unlike Yule-Nielsen, the expanded Murray-Davies model takes into account the varying micro-reflectance for the printed dots and the paper, thus providing a better physical description of optical dot gain in halftone reproduction. We have further shown that the characteristics of the varying micro-reflectance values for the dots and the paper depend strongly on the halftone geometry and the print resolution (Nyström, 2010). The variation of the micro-reflectance values for the ink and paper increases with increasing print resolution, and thus with decreasing dot size. For AM prints, the largest variations occur when size of the dots, and the paper area between the dots, respectively, becomes small. For FM prints, the variation of the ink and paper micro-reflectance is more linear with dot area coverage.

In this work, we continue the approach of micro-reflectance measurements of the halftone dots and the paper between them. The aim is to investigate if the approach can be further developed to also handle prints with multiple and overlapping colorants. We propose an expanded Neugebauer model,

incorporating the varying micro-reflectance for all the Neugebauer primaries, similar to the previously proposed expansion of the Murray-Davies formula. This approach requires for colorimetric measurements on a micro-scale level for the all the different combinations of printed colorants and the paper between them, as well as accurate methods for computing the physical dot area fractions for the Neugebauer primaries.

Experimental Setup

The Image Acquisition System

An experimental image acquisition system is used, specially designed for acquiring microscopic images of prints and substrates. The images are captured using a monochrome CCD camera, with a resolution of 1360×1024 pixels and 12 bit dynamic range. The CCD, specially designed for scientific imaging, is of grade 0, which means no defective pixels, and uses digital temperature compensation to reduce noise. It has previously been verified that the camera response is linear with respect to the intensity of the incident light (Nyström, 2008b). The optics used is a macro system, designed for scientific applications, allowing for images of various magnifications, up to a maximal resolution of $1.2 \mu\text{m}/\text{pixel}$.

The substrate is placed on a table, which allows for controlled translations in two directions and for rotation around the optical axis. The illumination is provided using a tungsten halogen lamp through optical fibers, which offers an adjustable and well-controlled angle of incidence, as well as the possibility of using a backlight setup. Color images are captured sequentially, using filters mounted in a filter wheel in front of the light source. By using this color sequential method, there is no need for any interpolation or de-mosaicing scheme, as is the case in conventional digital cameras. Besides the ordinary RGB-filters, the filter wheel also contains a set of 7 interference filters, which allows for the acquisition of multi-channel images. The image acquisition system has previously been thoroughly calibrated and characterized. Models have been developed, allowing for the device dependent images to be converted

into the colorimetric representations CIEXYZ and CIELAB, and also to reconstruct spectral reflectance data (Nyström & Kruse, 2006; Nyström, 2007).

Micro-reflectance Measurements of Halftone Prints

RGB and multi-channel images of the test prints have been captured using the 45°/0° measurement geometry. The field of view was 2.7×2 mm, giving a resolution corresponding to 2µm/pixel. All images are first corrected for dark current and CCD gain. The conversion to CIEXYZ color space was made by polynomial regression from RGB images, using characterization functions individually derived for each print (Nyström, 2007).

Printed Samples

The printed samples used in the study consisted of offset prints on a coated paper grade (150 gr/m²), printed with AM halftones, using clustered dots (100 lpi, 1200 dpi). The nominal dot area coverage of the patches are 0, 2, 5, 8, 10, 15,..., 90, 95, 98 and 100% respectively. The test charts were printed with a commercial 4-color offset press, and all the patches were printed using the same plate.

Macroscopic measurements of the spectral reflectance and the tristimulus values of the printed color patches were derived using a Gretag MACHBETH Spectrolino spectrophotometer, equipped with a UV filter, using the 45°/0° measurement geometry. All colorimetric computations were made using the CIE standard illuminant D65.

Methodology

An Expanded Neugebauer Formula

The aim of this work is to investigate if our previous approach of using micro-reflectance measurements of ink and paper to model halftone color reproduction can be further developed to also handle prints with multiple and overlapping colorants. Therefore, we propose an expanded Neugebauer model,

incorporating the varying micro-reflectance values for all the Neugebauer primaries, similar to the previously proposed expansion of the Murray-Davies formula, as:

$$R = \sum_j a_j R_j(a_j) \quad (5)$$

In this equation, the mean reflectance, R , is given by the sum of the reflectance of the Neugebauer primaries, $R_j(a_j)$, including overlapping colorants. The reflectance of the Neugebauer primaries will now depend on the dot area coverage, a_j , just as in the expanded Murray-Davies formula (Eq. 4). In this work, we will use this approach to measure and predict the CIEXYZ tristimulus. Thus, the reflectance values in Eq. 5 are replaced by the CIEXYZ tristimulus values, as:

$$\begin{aligned} X_{tot} &= \sum_j a_j X_j(a_j) \\ Y_{tot} &= \sum_j a_j Y_j(a_j) \\ Z_{tot} &= \sum_j a_j Z_j(a_j) \end{aligned} \quad (6)$$

where X_{tot} , Y_{tot} and Z_{tot} refer to the output tristimulus values (corresponding to the macroscopic color measurement), and $XYZ_j(a_j)$ is the micro-colorimetric values for the Neugebauer primaries. Naturally, in these equations, the sum of the area fractions, a_j , for the Neugebauer primaries will sum up to unity, as:

$$\sum_j a_j = 1 \quad (7)$$

By examining Eq. 6, we see that this approach requires for micro-colorimetric measurements of the all the different combinations of printed colorants and the

paper between them, $XYZ_j(a_j)$, for each area coverage, a_j . It will also require methods for computing the actual, physical dot area fraction for each of the Neugebauer primaries, which may differ from the nominal dot areas due to physical dot gain.

Micro-reflectance Measurements

To obtain the micro-colorimetric measurements, $XYZ_j(a_j)$, of each of the Neugebauer primaries, microscopic RGB and multi-channel images are captured of halftone prints. In microscopic images, i.e. when the spatial resolution of the images is high in relation to the resolution of the printed halftone dots, each individual halftone dot is portrayed in the image, allowing for micro-reflectance measurements of the halftone dots and the paper between them. The captured device-dependent RGB-images are first converted into CIEXYZ color space, using polynomial regression techniques (Nyström, 2007).

To capture the typical characteristics of a large population of halftone dots, which may differ in their appearance, 3D-histograms are computed from the microscopic images in CIEXYZ color space. A histogram is merely a probability density function for the occurrence of different colors in the image, and contains nominally no spatial information. However, with the a priori knowledge that the image represents a halftone print captured in micro-scale, one can relate properties of the histogram to spatial properties of the halftone, such as the edges of the halftone dots (Nyström & Kruse, 2005).

For images of halftone color prints captured in micro-scale, the only colors present are the ones representing the paper, the printed primary inks, and different combinations of overlapping primary inks. In a 3D color histogram from such images, the paper and the different combinations of printed inks will appear as clusters in the histogram, with the transitions between the clusters corresponding to the edges of the halftone dots. This requires the resolution of the images to be sufficiently high in relation to the resolution of the halftone, to ensure that each individual halftone dot is represented by a sufficient number of pixels, and that the characteristics of the edges are accurately represented in the

images. The micro-colorimetric values, i.e. the tristimulus values of the paper and the different combinations of ink, $XYZ_j(a_j)$, can then be computed as the centers of gravity of the clusters corresponding to the paper and ink, for each area coverage, a_j .

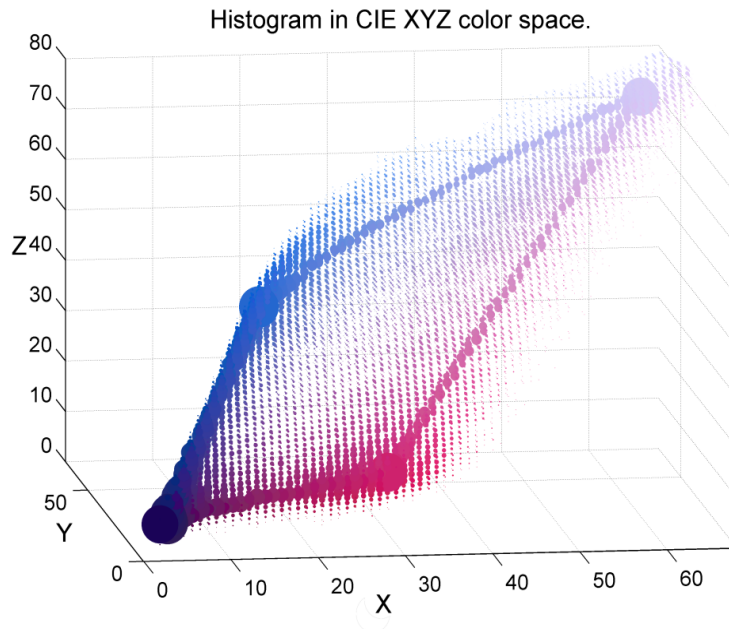


Figure 1. Visualization of a 3D histogram in XYZ color space for a 60% blue halftone print.

Figure 1 depicts an example of a visualization of a 3D color histogram in CIE XYZ color space, for a 60% blue halftone print (i.e., with the nominal dot area coverage 60% for cyan and magenta). The four clusters correspond to the paper, the primary inks cyan and magenta, and the secondary blue overlapping of the primary inks, i.e. the four Neugebauer primaries.

Finding the Physical Dot Area Fraction

A common assumption when computing the fractional areas for the Neugebauer primaries is that the halftone dots are randomly distributed on the substrate. The area fractions can then be calculated by the famous Demichel (1924) equations, based on the probabilities for a given point to be covered by a certain combination of inks. However, in this work it is important that the dot area fractions, a_j , for the Neugebauer primaries used in Eq. 6 corresponds to the real, physical, dot area fractions. Typically, the physical dot areas may differ from the nominal ones, due to physical dot gain in the printing process. Register shift, which may occur in the printing process, will further affect the relative fractional areas of the Neugebauer primaries (Gooran, et al., 2011).

To compute the physical dot area fractions will require for means of separating the physical dot gain from the optical dot gain. However, in reflection measurements of halftone prints the physical and optical dot gains always co-exist, making the separation of one type from another a difficult task. A comparison of different methods to separate physical and optical dot gain is given in Nyström & Yang (2009). For multiple color prints, the task will also include the separation of the different colorant inks.

Separation of the Colorant Inks

Beside the RGB images used in the conversion to CIEXYZ images, multi-channel images have been acquired of the same halftone patches. The multi-channel images make use of 7 interference filters, covering the visible spectrum, with the center wavelengths 400, 450, 500, 550, 600, 650 and 700 nm. Since the different inks absorb and reflect the light in different wavelength bands, we can use the multi-channel images to separate the different inks from each other (Namedanian, et al., 2011). For example, in an image of a blue print captured using the 700 nm interference filter, only the cyan ink will be visible in the image since this spectral region belongs to the absorbing wavelength of cyan, but to the reflecting wavelength band for the magenta ink. On the other

hand, an image captured using the 500 nm filter, will essentially depict only the magenta dots for the same reason.

This is illustrated in Fig. 2 for a 40% blue print (i.e. with 40% nominal dot area coverage for cyan and magenta). The upper half depicts a part of a microscopic RGB image of the print, showing the paper, the cyan and magenta halftone dots, and the overlapping blue. The lower left image shows a part of the same halftone patch captured using the 700 nm interference filter. Here, the magenta dots are not visible at all. The lower right image shows the image captured using the 500 nm filter, where the magenta ink is visible. Here the cyan halftone dots are still visible, but only as weak shadows, which are easily separated from the magenta ink.

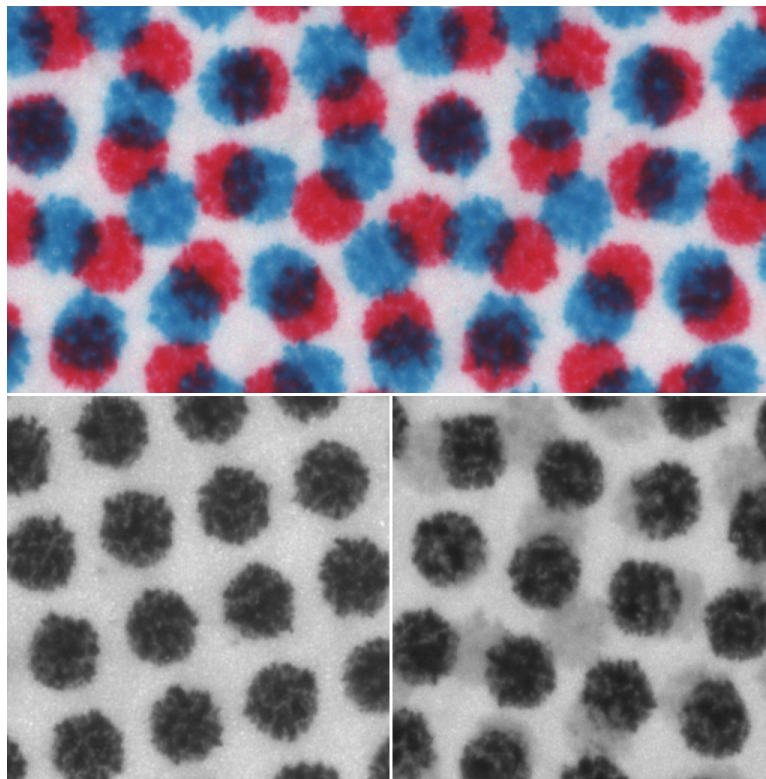


Figure 2. Separation of the cyan (lower left) and magenta (lower right) colorant inks.

Segmentation

After the different colorant inks have been separated into separate images, these images need to be segmented in order to compute the physical dot areas. This requires for a proper threshold value, defining the limit between the ink and the paper. When microscopic images of the prints are available, one possibility is to derive the threshold from image data. By using vertical and horizontal line scans across the halftone dots, the threshold value can be defined as the region of maximum rate of change in reflectance values, dR/dx (Arney, et al., 1995a). This method was used in our previous work, for single color prints (Nyström, 2008a). Another approach to compute the threshold value from histogram data of microscopic images was proposed in Namedanian & Gooran (2010). In this work, we make use of the thresholding algorithm proposed by Otsu (1979), minimizing the interclass variance of black and white pixels.

The resulting binary images after segmentation gives the physical dot areas for the cyan and magenta colorants. By combining the binary images, the area for the secondary blue is given where the cyan and magenta inks overlap. This is displayed in Fig. 3 for the same 40% blue halftone print as in fig. 2. Here, the binary images are used as masks in combination with the RGB images to display the physical dot areas for the 4 Neugebauer primaries paper, cyan, magenta and blue. The resulting physical dot area fractions for the full patch corresponds to 34.2% for paper, 23.7% for cyan, 22.2% for magenta and 19.9% for the secondary blue.

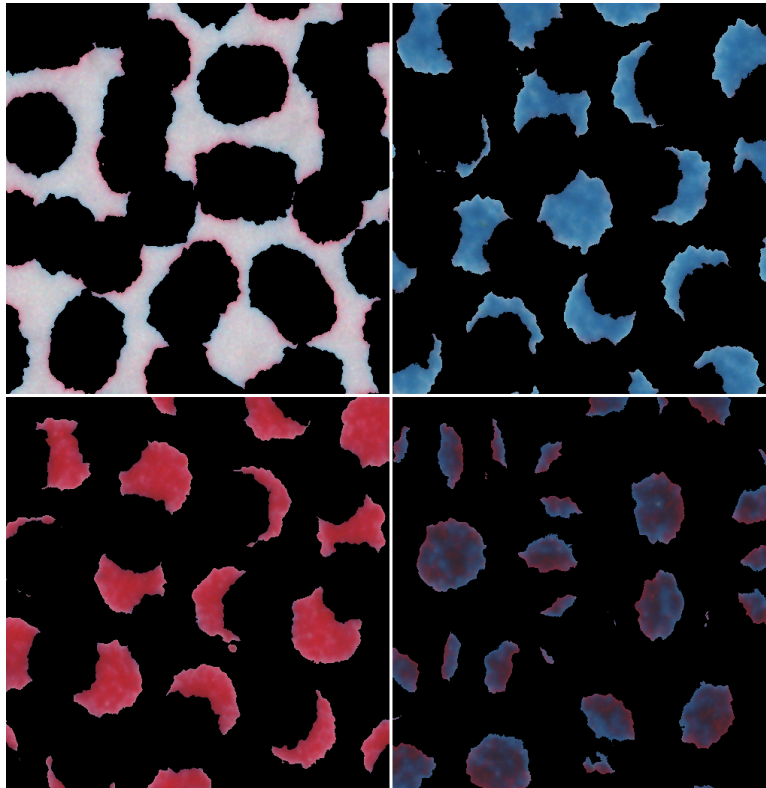


Figure 3. Physical dot areas for Paper (upper left), Cyan (upper right), Magenta (lower left), and Blue (lower right).

Experimental Results

The proposed methodology has been evaluated using printed AM halftones, with cyan and magenta inks. The nominal dot area coverage of the printed halftone patches are 0, 2, 5, 8, 10, 15, ..., 90, 95, 98 and 100%, using equal ink coverage for both cyan and magenta. The screen angles used correspond to 15° and 75°, for the cyan and magenta color separations, respectively. From the 24 halftone patches, the physical dot area fractions as well as the micro-colorimetric values,

have been derived according to the described methodology. The macroscopic colorimetric measurements of the printed color patches, used for comparison, were measured using a spectrophotometer.

Figure 4 displays the computed physical dot area fractions, a_j , for the 4 Neugebauer primaries paper, cyan, magenta and blue, with respect to the nominal dot areas a_0 (for cyan and magenta). The derived micro-colorimetric values, XYZ_j , for the Neugebauer primaries with respect to the nominal dot areas for cyan and magenta, are displayed in fig. 5. This figure makes it perfectly clear that there is a large variation for the measured micro-colorimetric values for the different combinations of ink, as well as the paper between the dots, depending on the total dot area coverage.

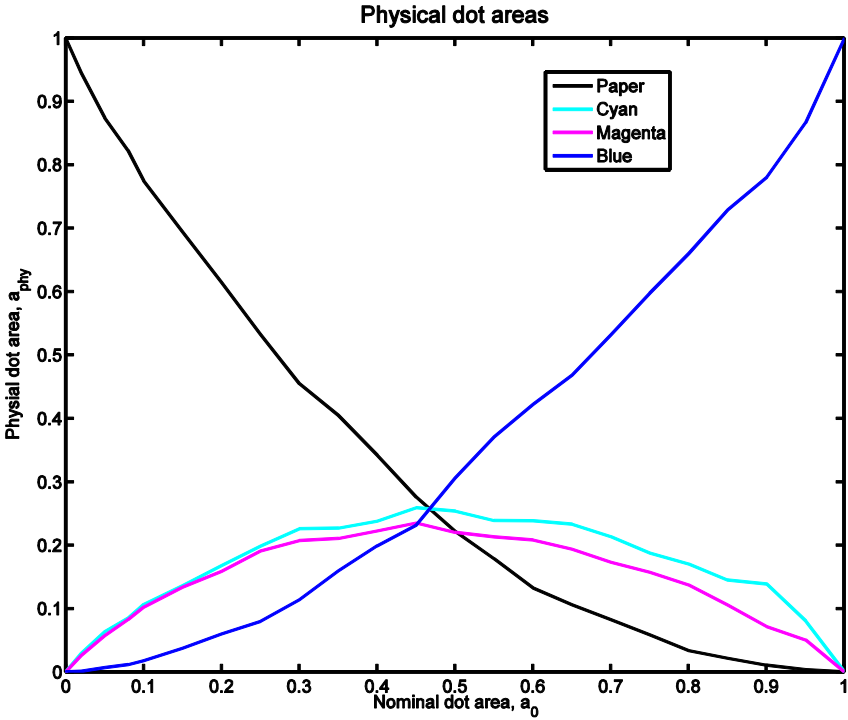


Figure 4. The computed physical dot area fractions for the Neugebauer colorants Paper, Cyan, Magenta, and Blue.

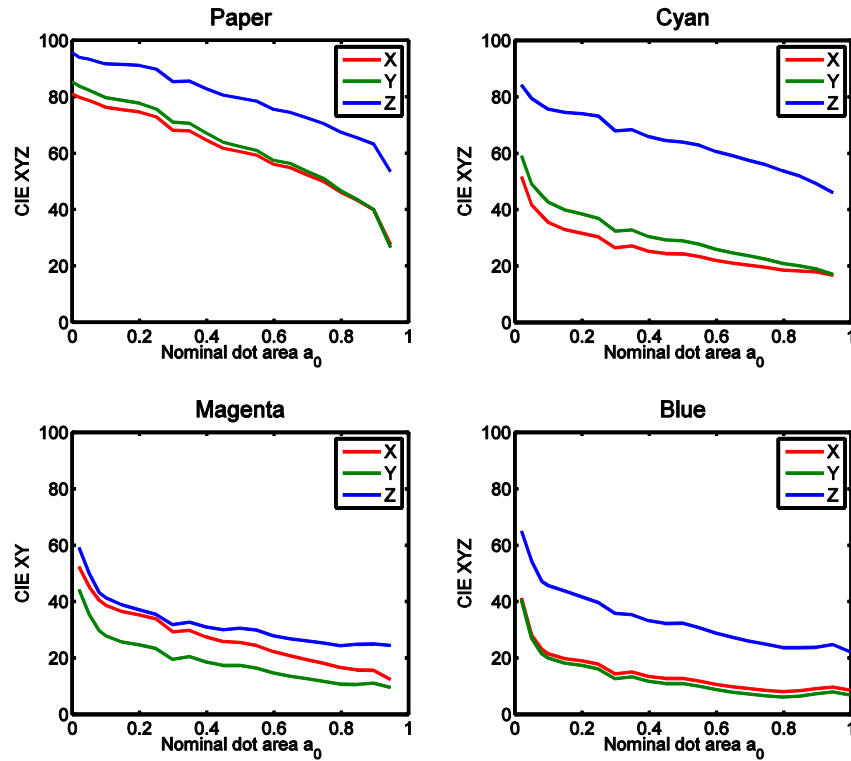


Figure 5. Micro-colorimetric measurements of the Neugebauer primaries
Paper, Cyan, Magenta, and Blue.

Figure 6 displays the result when combining the computed physical dot area fractions (Fig. 4) with the micro-colorimetric measurements of the Neugebauer primaries (Fig. 5), to compute the mean tristimulus values according the expanded Neugebauer formula (Eq. 6). Included as reference are the corresponding tristimulus values, measured by a spectrophotometer. Clearly, the tristimulus values predicted using the Expanded Neugebauer model are very close to the colorimetric measurements.

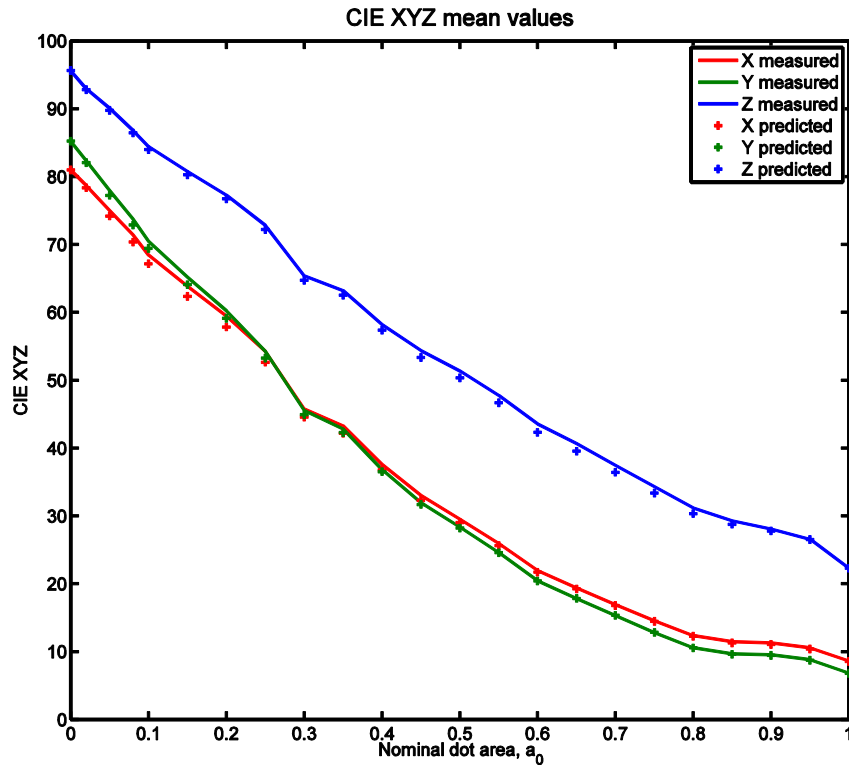


Figure 6. Measured and predicted colorimetric values.

The Euclidian distance in CIEXYZ color space, ΔXYZ , between the predicted and measured tristimulus values are given in Tab. 1, together with the CIE 1976 color difference, ΔE_{ab} . Notice that the values predicted according to Eq. 6 are computed from microscopic images of the prints, while the reference values used for comparison are measured by a spectrophotometer. This means that the color differences listed in Tab. 1 (and displayed in Fig. 6), also include the error from the conversion from RGB to CIEXYZ color space, for the microscopic images. Considering this, one must say that a mean prediction error of $1.2 \Delta E_{ab}$ and a maximum of $1.9 \Delta E_{ab}$ indicate that the proposed model is highly accurate.

Table 1. Estimation errors between measured and predicted tristimulus values.

ΔXYZ	ΔXYZ	ΔE_{ab}	ΔE_{ab}
max	mean	max	mean
2.01	1.04	1.87	1.20

Conclusions and Discussion

We have proposed an expended Neugebauer formula; incorporating the variation of the micro-reflectance of the Neugebauer primaries with respect to the dot area coverage. The methodology has been evaluated using offset prints of AM halftones with cyan and magenta colorants. The experimental results have shown that the method works well, giving only small prediction error. More importantly, the results have shown that the colorimetric variations for the Neugebauer primaries with respect to the total ink area coverage are substantial. These results clearly indicate that every model that assumes a constant reflectance of the inks and paper, including the ordinary Neugebauer as well as the Yule-Nielsen modified Neugebauer, are fundamentally wrong.

In future work, the proposed methodology should be evaluated also for other combinations of colorants, and for a greater number of different prints. We know from our earlier work on single color prints, that the way the micro-reflectance of the ink and paper vary with the dot area coverage is strongly related to the halftone geometry and the print resolution. Future work should be devoted to finding models for how the micro-reflectance of the ink and the paper varies with dot area coverage. Especially, focus should be on relating these functions to properties of the printing process, such as the paper substrate, the printing method and the halftoning employed. Microscopic color measurements of halftone prints will provide a powerful tool in future research devoted to gain a deeper understanding of the complex process of halftone color reproduction.

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